The role of spatial variability of soil moisture for modelling surface runoff generation at the small catchment scale

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The role of spatial variability of soil moisture for modelling surface runoff generation at the small catchment scale

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Abstract

The effects of spatial variability of soil moisture on surface runoff generation at the hillslope and small catchment scale were studied. The model used is physically based accounting for the relevant hydrological processes during storm runoff periods. A case study investigating the effects on runoff generation in a loessy small catchment is presented. In this study the storm rainfall response was modelled using different distribution patterns of the initial soil moisture content, and where different initial soil moisture fields were generated by using both interpolation methods and stochastic simulation methods. It is shown that spatial variability of pre-event soil moisture results in an increase in runoff production compared to averaged values. It is of particular importance to note the combined organised/stochastic variability features, that is, the superposition of systematic and random features of soil moisture dominate local generation of surface runoff. In general one can say that the stronger the organised heterogeneity is, the more important is an adequate and refined interpolation technique which is capable of accounting for complex spatial trends. The effects of soil moisture variations are of particular importance for storms, where the produced runoff volume is just a small fraction of precipitation.

Introduction: spatial variability and non-linear hydrological response

The spatial variability of catchment parameters or variables (e.g. topography, soil hydraulic functions, vegetation type, initial soil water content, surface roughness etc.) may have an important impact on describing rainfall-runoff processes, in particular under heavy rainfall conditions. A particularly high degree of spatial variability has been shown for natural earth materials at different scales in various field studies. For example, at the micro scale (a few m² to a hectare) Nielsen *et al.* (1973) give an example of variation of saturated conductivities, while Schiffler (1992) gives an example of the high variation of final infiltration rates, measured at 60 equidistant points within a 90 m long transect of loamy and sandy soil (Fig. 1), which is similar to the results of Gelhar (1984), who describes the high variation of infiltration rates of an irrigation area.

Lehmann (1995) documents the significant heterogeneity of soil moisture content under different hydrologic conditions of an agriculturally used loessy soil at a small catchment scale. Other examples of soil moisture heterogeneity are presented, e.g. in the work of Grayson *et al.* (1997), distinguishing different typical patterns for dry and wet catchment conditions, respectively. The soil moisture conditions (mean value and spatial variations) prior to a rainfall event play a central role in the generation of surface runoff by controlling the local infiltration capacity. However, it is difficult to quantify these effects, because the spatial variability of infiltration capacities and precipitation intensities results in a clear non-linear response of runoff to rainfall. As explained by Beven (1995), this non-linearity response arises due to the coupling of different hydrological processes, which in the case of the present study is the coupling of infiltration and surface runoff generation. To demonstrate this property consider the following simple example:

The runoff, $Q(N)$, at a given point x, determined by precipitation, $N(x)$, and infiltration capacity, $\Phi(x)$, is calculated using:

$$Q(N,\Phi = \begin{cases} 0 & \text{if} \quad N < \Phi \\ B - \Phi & \text{if} \quad N \geq \Phi \end{cases} \tag{1}$$

Supposing one calculates the total runoff, Q_A, over a given area A the response depends on the spatial distribution of N and Φ.

$$Q_A = \int_A Q(N(x), \Phi(x))\, dx \tag{2}$$

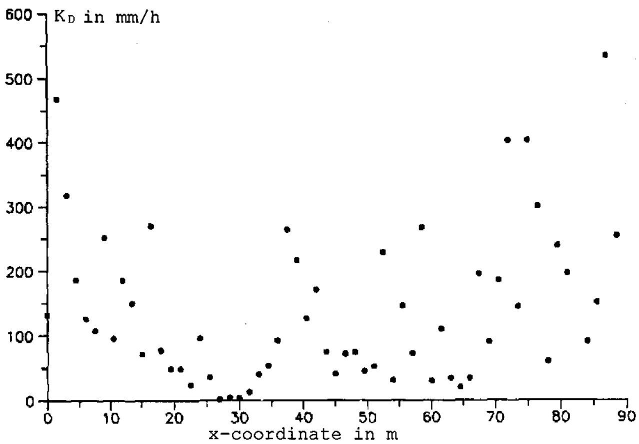

Fig. 1. *Final infiltration rate k_D of grassland on a loamy-sandy soil in the upper Rhine valley, Germany; measurements were made by double ring infiltrometers at 1.50 m intervals at 60 locations within a 90 m transect (from Schiffler, 1992).*

Q_A can be calculated for different spatial distributions of N and Φ. Supposing N is constant over A this is:

$$Q_A = \int_{N>\Phi(x)} N - \Phi(x)\,dx = P(N > \Phi(x))[N - \bar{\Phi}_N] \qquad (3)$$

with $\bar{\Phi}_N = \int_{N>\Phi(x)} \Phi(x)dx$ being the mean infiltration capacity over the area with N exceeding $\Phi(x)$. One can see that due to the convexity of the function defined in Eqn. (1) the point function used with the areal mean values leads to an underestimation of Q_A. If the heterogeneity of N is also included then the effects are similar. In this example Q_A was calculated for different distributions of infiltration capacity. Figure 2 shows Q_A as a function of N for different Φ distributions. The thick line represents the homogeneous case, the thin line the heterogeneous case. One can see from the figure that the non-linearity of the

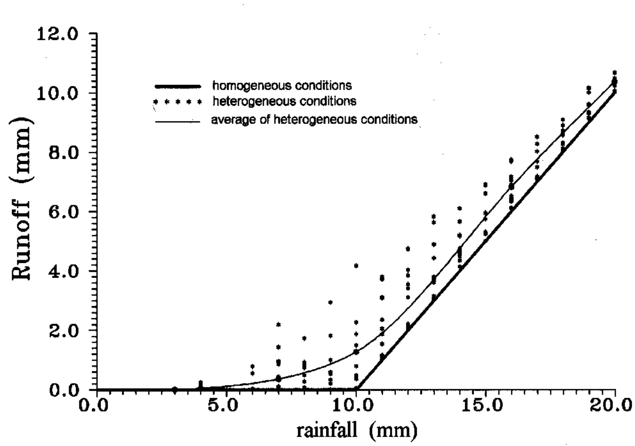

Fig. 2. *Principal scheme of linear and non-linear response to rainfall.*

response implies that for heterogeneous conditions the response is always above the homogeneous case (which also represents the mean values). The differences are the largest around the value $\bar{\Phi}_N$ (in our example 10 mm) and decrease with increasing rainfall.

The effects of spatial variability in hydrology have been studied for different kinds of variability with a focus on various hydrological processes. Woolhiser (1986), Woolhiser and Goodrich (1988), and Faurès *et al.* (1995) have pointed out the impact of spatial and temporal rainfall variability on Hortonian runoff production in semi-arid watersheds. Sharma and Luxmoore (1979) presented a simulation study of the effects of soil hydraulic property variability on water balance modelling, while Woolhiser *et al.* (1996) investigated the effects of the spatial variability of saturated hydraulic conductivity, again on Hortonian runoff production in a semi-arid environment. The paper presented here concentrates on the effects of spatial variability of soil moisture.

Both Seyfried and Wilcox (1995) and Blöschl and Sivapalan (1995) distinguish spatial variability of soil moisture composed of a stochastic and of an organised (or deterministic) component. Western *et al.* (1996) mentioned that the latter is the result of and is reflected by the long-term geo-eco-morphological evolution of the specific landscape. For example, at the hillslope scale Beven (1995) reports that *the pattern of hillslope forms and related soil characteristics might be important in controlling the variability in responses of hillslopes and small catchments.*

The first purpose of this paper is to evaluate whether and to what extent the effects of spatial variations of soil moisture are realised in the hydrological response to heavy rainfall in a real catchment. In order to concentrate on runoff generation processes only (not to consider channel routing processes) the scale chosen is that of a small catchment, i.e. of a few hectares. Additionally, by choosing this scale, it was possible to neglect the effects of the spatial variations of rainfall intensities.

When using spatial distributed catchment properties as input into a distributed hydrological model, it is necessary to apply an interpolation technique to derive an estimate of the spatial pattern of the specific parameter. Taking the soil moisture content as an example, it is the second purpose of this paper to investigate the influence of choosing different patterns of organised variability (as a result of applying different interpolation methods) on the modelled runoff response of a small catchment.

The study presented here also aims to complement previous work, such as Grayson *et al.* (1995), who compared runoff modelling results for a micro-catchment ($2m^2$) with both organised and stochastic initial soil moisture conditions, and Merz and Plate (1997), who investigated the scale dependency of runoff generation as a function of the organised and stochastic variability of soil conductivity and event size. So the third purpose of this paper is to investigate the effects of the stochastic component of soil moisture variability and to compare the influence of the organised and the stochastic component.

Methods applied

This study is done by a modelling analysis of the storm runoff conditions in a small headwater catchment considering varying heterogeneity of initial soil water content. The model applied has been specifically developed for detailed analysis of storm runoff processes at the scale of hillslopes or small catchments. The necessary pre-event soil moisture fields are based on several point measurements of soil water, with its spatial distribution pattern being generated with both interpolation methods and stochastic simulation methods.

PHYSICALLY BASED HYDROLOGICAL MODELLING AT THE SCALE OF HILLSLOPES AND SMALL CATCHMENTS

The model used for this study is the physically-based, distributed hydrological model at the hillslope and small catchment scale 'Hillflow'. Three versions of this model are available: a one-dimensional (vertical) version, a two-dimensional (horizontal/vertical) version and a quasi three-dimensional version, which enables simulation of the hydrological processes in a small catchment of any shape.

The model simulates a series of hydrological processes such as interception, evaporation from the soil and canopy, plant transpiration, infiltration into the soil matrix and into macropores, water dynamics in the soil matrix, non-Darcian subsurface stormflow including possible reappearance to the surface (return flow) and surface runoff. The relevant process interactions are included and the flow processes in the soil are considered in both vertical and lateral directions. Special regard has been given to the consideration of rapid soilwater flow processes during storm conditions, which is summarised below. The soil is considered to be composed of a double porous domain: the soil matrix (or micropores) and the macropore system. The macropore system has been implemented to reflect the voids, large pores etc. near the soil surface (e.g. due to biological activities in the root zone or due to tillage), so the depth of the macropore layer, H_Z, is a model parameter to be prescribed by the model user and in general extends a few dm. The whole model is described in detail by Bronstert (1994).

The *infiltration* rate, I, is considered to be composed of the two sub-processes infiltration into the micropores (controlled by the matrix potential) I_{mic} and infiltration into the macropores (controlled by the macropore volume) I_{mac}.

$$I = I_{mic} + I_{mac} \tag{4}$$

By this approach both slow infiltration for low intensity rain and rapid infiltration for high intensity rain storms (possibly bypassing the soil surface) are taken into account. The infiltration component into the macropore system is switched on when the net precipitation rate, I_{Nnet}, exceeds the micropore infiltration rate. The water within the macropores and the moisture in the micropores, I_{mic}, interact, i.e., the macropore water may (partially) infiltrate into the soil matrix as long as the average moisture value of the matrix within the macropore layer, H_Z, is below saturation. This interaction is approximated by a simple non-saturated Darcy approach. I_{mic} is calculated according the theory of potential flow (Richards-Equation) and I_{mac} is estimated by a simple bucket approach, where the bucket drainage rate is the macropore water infiltrating into the surrounding matrix. It has been shown by Bronstert and Plate (1997) that high infiltration rates often observed in natural conditions can be modelled by this approach, without prescribing high (i.e. unrealistic) values for the hydraulic matrix conductivity.

The model also represents runon infiltration; runoff generated at an upper part of the hillslope (2-dimensional case) or catchment (3-dimensional case) may partly infiltrate on its way downslope according to the respective local saturation and conductivity conditions.

The computation of *surface runoff* is based on the assumption of sheet flow. For the three-dimensional model version, a quasi two-dimensional approach for overland has been applied, i.e. all orthogonal and diagonal adjacent grids of lower elevation are principal recipients of surface runoff from the grid under consideration. This distribution approach is explained in more detail by Bronstert (1994). For the two-dimensional model version, the overland process flow is simplified by a one-dimensional cascade of constant width, as proposed by Kibler and Woolhiser (1970). The diffusion analogy (considering gravity, friction and pressure forces) is applied, consisting of the continuity equation,

$$\frac{\partial q}{\partial x} + \frac{\partial h}{\partial t} - i = 0 \tag{5}$$

where q is the specific runoff per unit width, x the coordinate in flow direction, h the flow depth, t the time coordinate and i the effective inflow rate, i.e. the rate of infiltration excess $I_{Nnet} - I$ and the simplified momentum equation:

$$S_f + \frac{\partial h}{\partial x} = S_0 \tag{6}$$

where S_0 is the surface slope and S_f the friction slope, which is calculated according to the Manning-Strickler equation.

The possible lateral conduction of *subsurface stormflow* can be, in addition to the increase of soil infiltration capacity, a second important effect of a macroporous soil. This quick transmission of soil water downslope occurs when the macropore system has a sufficient degree of connectivity and when there is a significant infiltration into the macropore system. Subsurface stormflow is represented in the model by a non-Darcian flow approach within a soil layer of depth H_Z parallel to the ground surface which is analogous to the computation for surface runoff:

$$\frac{\partial q_Z}{\partial x} + \frac{\partial h_Z}{\partial t} - i_Z = 0 \tag{7}$$

where q_Z is the specific subsurface stormflow per unit width, h_Z the water content in the macropore layer (subsurface flow depth), and i_Z the inflow rate to the subsurface cascade.

The determination of the inflow rate to the subsurface cascade, i_Z, requires a water budgeting in the macropore layer (H_Z), i.e., calculating the difference between macropore infiltration, I_{mac}, and the flux from macro- into micropores, $I^Z{}_{mic}$,

$$i_Z = I_{mac} - I^Z_{mic} \tag{8}$$

Two options to calculate the lateral subsurface stormflow velocity have been implemented:

- the subsurface stormflow velocity, v_Z, being essentially a linear function of the slope S_0 (inter flow for unconfined-saturated subsurface conditions according to Eagelson, 1970):

$$v_Z = S_0 \cdot k_Z \qquad (v_Z < S_0) \tag{9}$$

where the parameter, k_Z, is termed 'interflow conductance', averaging the lateral near-surface energy losses

- v_Z being essentially a non-linear function of the interflow conductance and the subsurface flow depth (lateral subsurface macropore flow surrounded by an absorbing matrix according to Germann, 1990):

$$v_Z = S_0 \cdot k_Z \cdot h_Z^{1.5} \qquad (v_Z < S_0 \cdot h_Z^{1.5}) \tag{10}$$

If the subsurface flow depth h_Z (i.e. the water content in the macropore system) exceeds the available macropore space (i.e. the product of macropore volume and depth of macroporous layer, $V_{mak} \cdot H_Z$), then the excess volume is assumed to return to the surface as return flow, yielding an increase in the surface water depth.

The model approaches for the processes of interception storage, plant transpiration, soil and canopy evaporation (Penman-Monteith formula) and unsaturated matrix moisture dynamics (theory of potential flow) are represented in the model by standard methods, which will not be repeated here. Groundwater flow, snowmelt and surface sealing effects (e.g. freezing, crusting) are not covered by the model.

This model was selected for our study because it is fairly comprehensive, allowing a fine discretization and parameterisation in time and space. Figure 3 gives a schematic representation of the quasi three-dimensional version and lists the processes considered. Bronstert and Plate (1997)

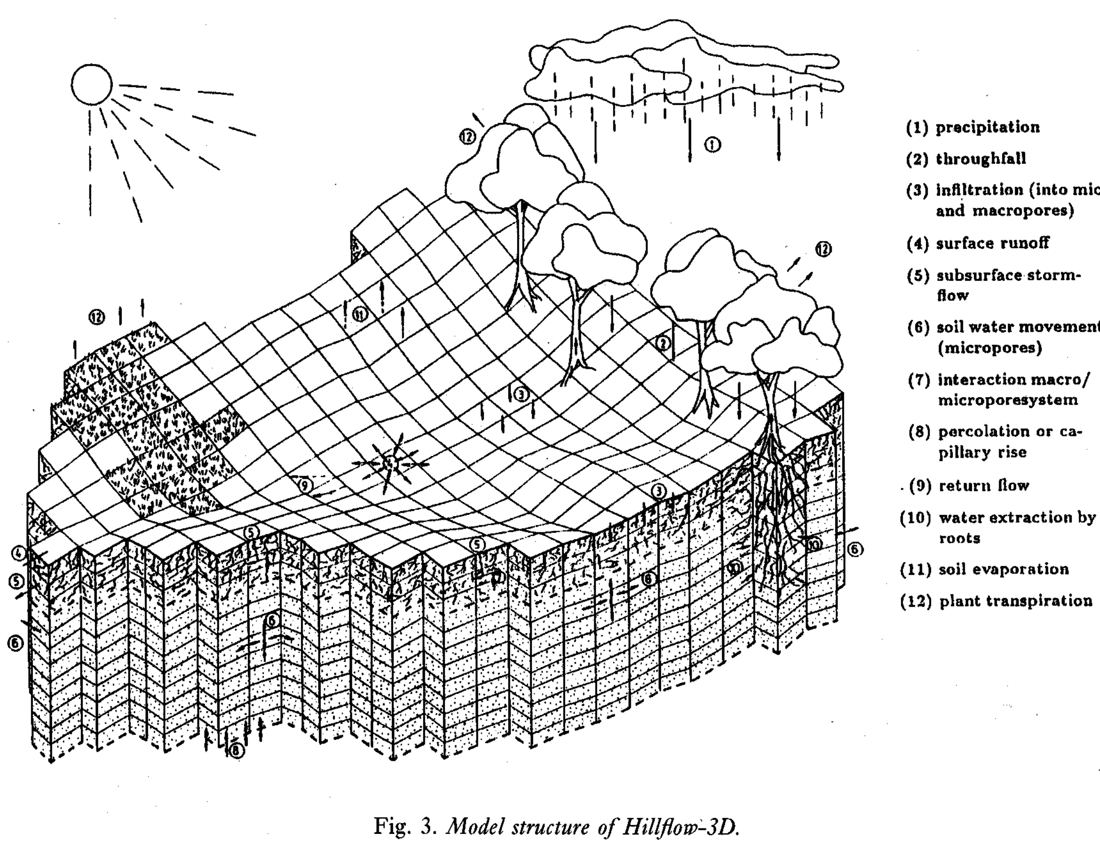

Fig. 3. *Model structure of Hillflow-3D.*

give an overview of the modelling approaches adapted and included in the model. Bronstert (1999a) presents several applications of the model for 1-, 2- and 3-dimensional simulations in the area of hillslope hydrology and environmental technology, and discusses the principle application limits.

THE PRE-EVENT CATCHMENT CONDITIONS: THE SPATIAL DISTRIBUTION OF SOIL MOISTURE

In many cases, soil moisture is measured at selected locations, but in fact, is needed for the entire catchment. Therefore a spatial 'extension' of the point values is necessary. This can be done by an interpolation method or a simulation technique. Interpolation methods produce estimates with relatively small mean errors—and thus necessarily smooth surfaces. Simulation methods produce 'alternative realities' which have the spatial variability of an observed case. If the non-linearity of the process is important, then simulations have to be used to calculate the expected response.

Geostatistical methods belong to the most powerful methods to interpolate spatial observations. For this study two techniques were used: ordinary kriging and Bayes-Markov updating. Among the methods which use the measured data only, ordinary kriging (OK) Matheron (1971) delivers the spatial distribution with a minimum estimation error. External knowledge having an influence on the soil moisture, such as topography or land use related parameters, can be incorporated using Co-Kriging or External Drift Kriging (EK) (Ahmed and de Marsily, 1987). If the additional information is conditioned, then Markov-Bayes Kriging (MBK) (Zhu and Journel, 1993) gives an alternative. The advantage of this method is that the influence of the external information is considered specifically for the actual catchment conditions at the observation time and may even vary in space and time. Bárdossy and Lehmann (1998) compared different soil moisture interpolation methods using a cross validation approach. They found that the simplified version of MBK, the Markov-Bayes Updating (MBU) using the topographic index $ln(a/tan\beta)$ (Beven & Kirkby, 1979) delivers the estimator with the smallest squared error.

Interpolation methods always deliver a smoothed image of reality. In order to obtain realisations with the observed stochastic variability, geostatistical simulation methods—such as turning bands or sequential simulation—have to be used. They can be used either directly or after an interpolation simulating possible deviations from the interpolated values. In order to consider the best interpolation method as a basis for the stochastic realisations of the spatial fields deviations from the BMU surface were simulated. To relate it to the measurements the deviations at their locations were considered to be zero.

A case study: storm runoff simulation at the small catchment scale

As an application example, the whole procedure was performed for a heavy rainstorm event at a small agricultural catchment in Southwest Germany, where detailed discharge measurements, soil moisture values and meteorological data were available.

CATCHMENT DESCRIPTION

A case study is presented, investigating the effects of spatial variations of soil moisture. The simulated area is a subcatchment of the 6.3 km² Weiherbach Catchment, an agricultural catchment of deep loessy soil located in the hilly Kraichgau region in south-western Germany. The parent soil material is loess, while three sub-types (pure loess, alluvial loess and loamy loess) can be distinguished. Its topography is characterised by a gently rolling landscape with altitudes varying between 140 m+asl and 240 m+asl.

The Weiherbach Catchment has been the subject of a comprehensive and multi-disciplinary research project to study flow and matter transport in rural catchments. A variety of hydrological, meteorological, soil-physical and soil-chemical measurements, as well as detailed monitoring of vegetation, land use and agricultural in- and output has been performed, see Fig. 4. More then 90% of the area is used as arable land, the remaining parts are forested or have few farmer settlements. For a further description of the catchment and the collected data see Plate (1992).

AVAILABLE DATA

In this case study the quasi three-dimensional model version of Hillflow has been applied on the 31.2 ha headwater catchment 'Neuenbürger Pfad' within the Weiherbach (see Fig. 4). For this area topographical, land cover and soil type data are available in a 12.5 m grid resolution. The catchment has been discretised into 1997 areal grids (12.5 m × 12.5 m each) and into 20 soil layers (5 cm each), yielding a total number of 39940 soil elements. The depth of the macropore layer has been assumed to be prescribed by the tillage depth, which was about 35 cm, i.e. the model's upper seven soil layers contain both matrix and macropores. The area is equipped with a discharge gauge at the outlet and a meteorological station nearby.

The catchment rainfall was measured at this climate station, about 700 m from the catchment in 5 min intervals, which was adopted as the standard input interval for precipitation in this modelling study. Because the variations of rainfall in space are not studied in this paper, the measured values were assumed to be representative for the whole modelled catchment. Spatial rainfall measurements in the Weiherbach catchment during the years 1992 to 1994, Kolle and Fiedler (1995), supported this assumption:

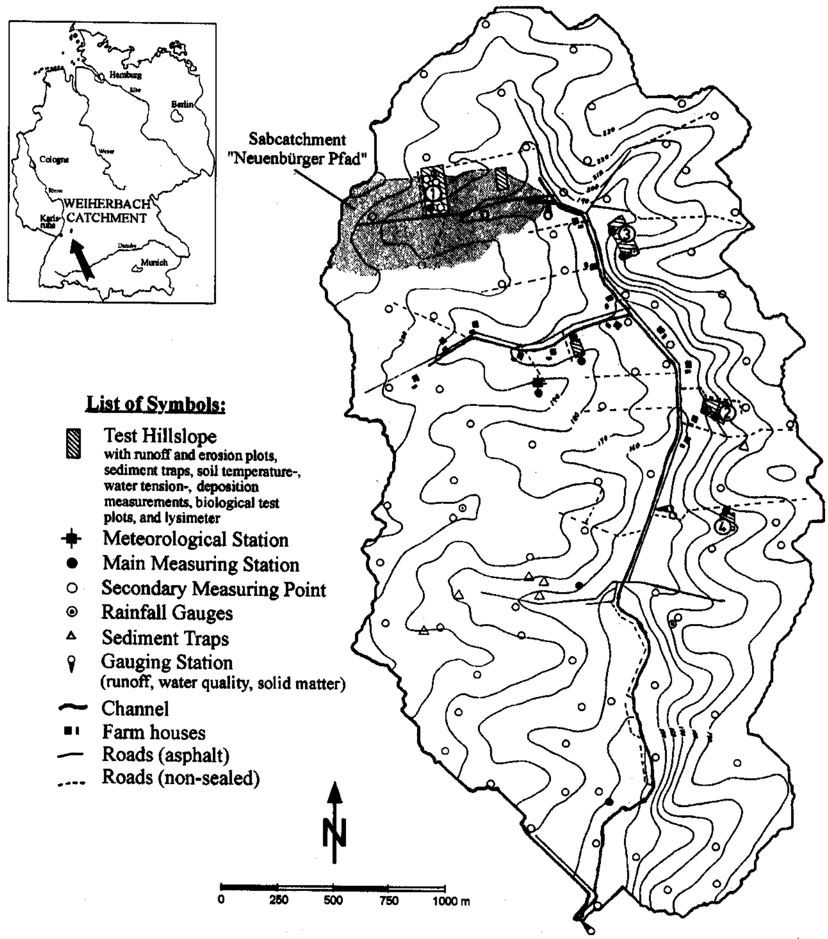

Fig. 4. *Location of the Weiherbach Catchment and experimental equipment installed. The sub-catchment Neuenbürger Pfad is marked in the North-western part.*

The annual precipitation values of 5 (out of 7) pluviographs in the catchment varied by less the 5% from the climate station. The remaining two rainfall stations (being more distant from this sub-catchment) showed a higher deviation (15% smaller and 6% higher value) which is explained by the specific leeside/windside exposure of these two stations. They also showed that the differences of rainfall measured at different pluviographs for specific events were small.

During the years 1991–1996 soil moisture measurements were recorded weekly (and partly bi-weekly) at about 100 points and 4 depths (0–15 cm, 0–30 cm, 0–45 cm, and 0–60 cm) in the whole Weiherbach Catchment using Time Domain Reflectometry (TDR). This yields a total of about 50 000 single soil moisture measurements.

The case study was made by modelling the 2-week period July 13–27, 1992. Within that period, on July 21, a heavy convective rainstorm appeared, with a total rainfall of 33.4 mm in 90 minutes, while the maximum measured intensity during an interval of 5 minutes was 98.4 mm/h between 6.25pm and 6.30pm. The measured total runoff volume was 109.6 m^2 (or 0.35 mm), i.e. the event had a runoff coefficient of 1.05%, which is a rather small value but quite typical for this area with deep loess soil. The peak discharge was 77.2 l/s, measured at 7.30pm. The soil moisture measurements used for the simulations were based on the 18 soil measurement points located in the subcatchment 'Neuenbürger Pfad'. The measurements at these points (at the 4 depths, see above) on the first day of the simulation period, July 13, 1992, have been used as the basis for the interpolation and the subsequent simulation procedures to obtain the different possible distributions of the pre-event soil moisture fields. The measurements of the last day of the simulation period (i.e. July 27, 1992) were used as verification of the soil water content calculated by the model.

Table 1 summarises the variables and the parameters used in the hydrological model. These parameters have been derived from the various measurements obtained in the catchment.

Table 1. Variables and parameters used for the case study

variable	parameter [unit]	value used in the case study	spatially distributed	temporal resolution
saturated conductivity	k_s [mm/h]	pure loess: 3.0 alluvial loess: 1.5 loamy loess: 1.0	areal zoning	no
saturated water content	θ_s [%]	pure loess: 0.40 alluvial loess: 0.43 loamy loess: 0.45	areal zoning	no
initial water content	θ_{ini} [%]	measurement base: from 18 TDR locations, 4 depths	interpolated and generated distribution	no
soil surface roughness	Strickler Coeff. [$m^{1/3}/s$]	2.0	constant	no
macroporosity	effect. depth H_Z [m]	0.35	constant	no
	m-pore space [Vol-%]	1.20	constant	no
vegetation	LAI[-] root depth [cm] interception [mm] plant lenth[m]	0.5–2.7 40–100 1.5–2.3 0.2–20	vegetation zoning	weekly
meteorological data	rainfall [mm]	(as measured)	constant	5 min
	net radiation [W/m^2]	—"—	—"—	1 h
	air temperature [C]	—"—	—"—	1 h
	wind velocity [m/s]	—"—	—"—	1 h
	rel. humidity [%]	—"—	—"—	1 h

Results

INTERPOLATION AND GENERATION OF SPATIAL SOIL MOISTURE DISTRIBUTIONS

The initial soil moisture conditions were derived for each model grid from the measured point data (18 measurement locations in the area) by the interpolation or simulation methods mentioned before. The effects of four different cases of initial soil moisture estimates have been investigated:

1. constant initial soil moisture contents for the whole area (but variations in depth are possible);
2. interpolated initial soil moisture values by Ordinary Kriging (OK);
3. interpolated initial soil moisture values by Markov-Bayes-Updating (MBU);
4. simulated initial soil moisture values using the MBU interpolation result as a starting point and superimposing a stochastic component. The degree of stochasticity is derived from the representative variogram based on all measured values of the Weiherbach Catchment, see Lehmann (1995). 20 realisations of these organised/stochastic soil moisture fields have been produced for the chosen date, conditioned on the measurements taken at the 18 locations of this subcatchment.

The first case reflects no spatial variability in soil moisture, cases 2 and 3 reflect the organised component of soil moisture heterogeneity (while in case 3 the topographic influence has been included into the interpolation scheme), and case 4 reflects both the organised and stochastic component. This procedure has been performed for 4 soil depths each 15 cm thick, i.e. for the layers 0–15 cm, 15–30 cm, 30–45 cm, and 45–60 cm.

As time series are available at each point variograms were calculated simultaneously, assuming a time invariant variogram (up to a multiplicative factor). A time invariant normed variogram was selected as follows. Suppose the variogram at time t is defined as:

$$\gamma_t(h) = \mathrm{Var}_x[Z(x+h,t) - Z(x,t)] = \tfrac{1}{2}E[(Z(x+h,t) - Z(x,t))^2] \tag{11}$$

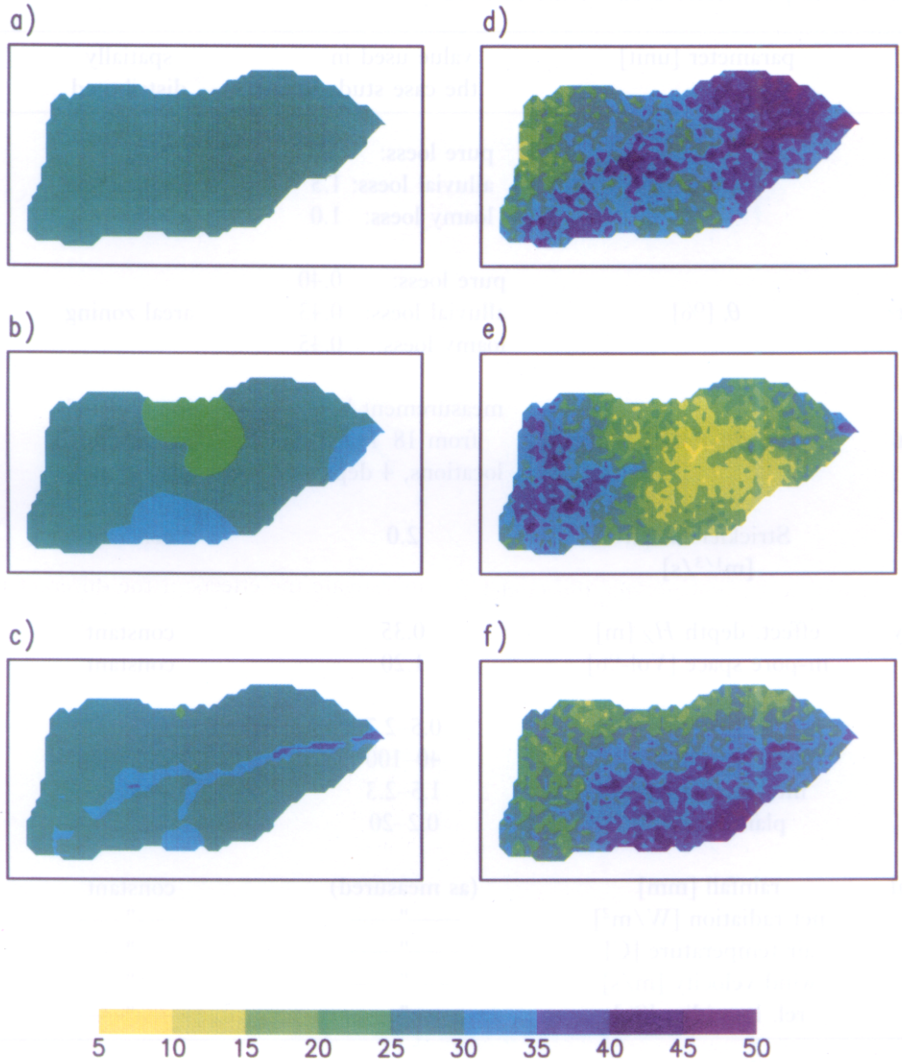

Fig. 5. *Initial soil moisture values (in Vol-%, averaged over the depth 0–60 cm) at July 13, 1992, subcatchment Neuenbürger Pfad, obtained by different methods: a) area averaged value; b) by the OK method; c) by the MBU method; d)–f) three realisations of the simulation technique.*

It is assumed that a common variogram, $\gamma(h)$, exists such that:

$$\gamma_t(h) = \mathrm{Var}_x[Z(x+h,t) - Z(x,t)] = \mathrm{Var}_x[Z(x,t)]\gamma(h) \qquad (12)$$

The time invariant variogram was estimated as:

$$\gamma^*(h) = \frac{1}{2KN(h)} \sum_{k=1}^{K} \sum_{|x_i - x_j|} \frac{(Z(x_i,t_k) - Z(x_j,t_k))^2}{Var(Z(x,t_k)} \qquad (13)$$

Here $N(h)$ is the number of pairs of measurement locations separated approximately by a vector h.

The interpolations were carried out using a spherical theoretical variogram fitted. The variogram was rescaled by the observed variance for the given time of the measurements. The variogram $\gamma_t^S(h)$ used for the turning band simulation of the deviations was obtained by taking:

$$\gamma_t^S(h) = (\mathrm{Var}[Z(x,t)] - \mathrm{Var}[Z^*(x,t)])\gamma(h) \qquad (14)$$

$Z^*(x,t)$ being the interpolated values (OK or MBU) calculated all over the catchment. Assuming that the interpolated surface and the interpolation error are independent, this approach assures that the simulated and observed variabilities are the same. In order to take the interblock variability into account a conditional block simulation was carried out.

Figure 5 shows the initial soil moisture values in Vol-% at July 13, 1992 (depth averaged over all layers: 0–60 cm) obtained by the different methods mentioned above. Figure 5a represents the area averaged value (27.7 Vol-%), Fig. 5b the values obtained by the OK interpolation method, Fig. 5c the values by the MBU interpolation method and Figs. 5d, 5e and 5f three realisations obtained by the simulation technique including both organised and stochastic hetero-

Table 2. Peak discharge Q_{max}, runoff volume V and duration of runoff D_r for subcatchment Neuenbürger Pfad, calculated by using different interpolated initial soil moisture fields

	areal constant	interpol. by OK-method	interpol. by MBU-method	measured values
Q_{max} [l/s]	24.6	97.7	75.5	77.2
V [m³]	38	152	115	103
D_r [min]	66	89	81	130

geneity. Figure 5d represents a realisation which is rather wet in average, Fig. 5e in contrast shows a realisation rather dry in average and Fig. 5f shows a realisation with an average wetness close to the interpolated average.

HYDROLOGICAL MODELLING

The most important (sensitive) parameter for the modelling of storm runoff generation are the soil parameters, such as hydraulic conductivity k_s, saturated water content θ_s, and effective macropore depth H_Z. As a first estimate, these parameters have been derived directly from measurements, field observations, or previous hillslope modelling analysis in that area. Further calibration steps (trial-and-error method to optimise the measured discharge hydrograph at the outlet) were performed by varying the soil parameters within rather tight limits, i.e. within less then 15% derivation from the respective starting value.

The initial soil moisture condition obtained by the BMU-method (Fig. 5c) being the interpolation including the topographic influence has been used as the reference case, i.e. this distribution pattern was used during the calibration procedure. It was considered as the reference case because this interpolation method is the one with the

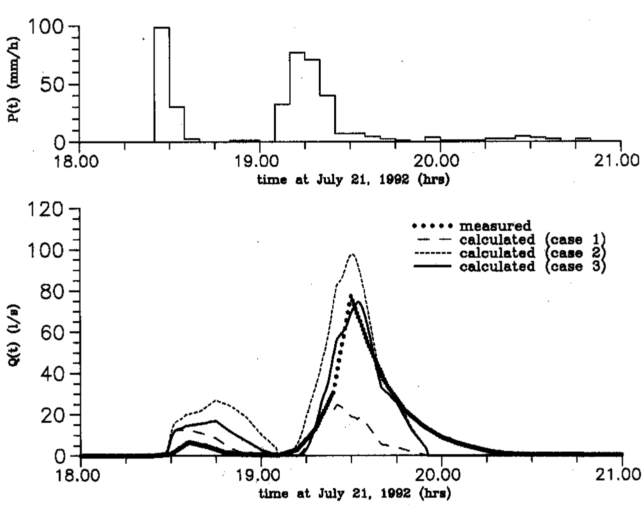

Fig. 6. *Simulated runoff hydrographs from subcatchment Neuenbürger Pfad, obtained by using different interpolated initial soil moisture fields (storm event of July 21, 1992)*

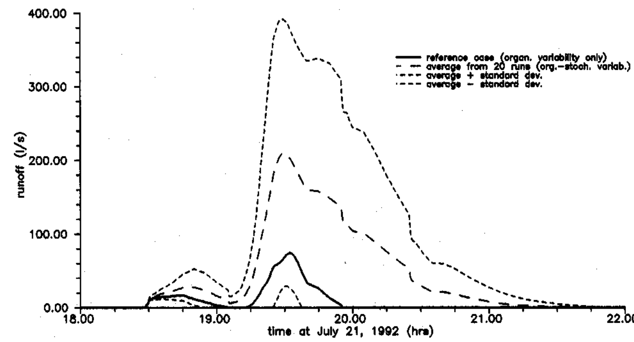

Fig. 7. *Simulated hydrographs and their standard deviation from subcatchment Neuenbürger Pfad, obtained by using organised and 20 realisations of organised/stochastic initial soil moisture fields (storm event of July 21, 1992)*

smallest estimation error, as shown by Lehmann (1995). After the calibration procedure, Hillflow was applied to investigate the effects of the different initial soil moisture fields. The remaining parameter-sets remained unchanged.

Figure 6 shows the results of the calculated hydrographs with the three different interpolation methods as the basis for the initial soil moisture conditions and compares them to the measured values. The response of the catchment is reflected in a rather satisfactory manner. However, the first peak is a bit overestimated and the falling limb of the main peak is too steep. This might be due to inaccurate representation (or limited knowledge) of some hydrological processes and their dynamic evolution during the event, such as litter and hollow storage, and runoff concentration (and recession) in flow paths on the catchment surface. Table 2 gives an overview of the obtained runoff volumes and peak discharges. The graphs show that the differences in modelled runoff for the different ways to describe the organised variability are notable but not extreme (maximum discharge due to the MBU interpolation 75.7 l/s, and due to the OK interpolation 97.7 l/s). These rather moderate differences are due to the fact that the variability of soil moisture at July 21 did not show a strongly organised pattern, i.e. the differences in interpolation methods do not lead to very different pictures of initial soil moisture conditions, as shown in Fig. 5b and Fig. 5c. However, it can be seen that the areal averaging leads to a significant underestimation of runoff. This phenomenon is

Table 3. Peak discharge Q_{max}, runoff volume V and duration of runoff D_r for subcatchment Neuenbürger Pfad, calculated by using 20 realisations of simulated/interpolated initial soil moisture fields

	mean value	min. value	max. value	standard deviation	coeff. of variation
Q_{max} [l/s]	227.7	15.26	716.75	186.4	0.82
V [m³]	638.5	21.59	2825.96	690.6	1.08
D_r [min]	131.4	66	218	46.6	0.35

explained by the fact that the areal averaging wipes out the small values of saturation deficit of some wet local sub regions, hence the number of grids producing infiltration excess is less pronounced.

Figure 7 compares the results of the simulated hydrographs obtained from different realisations of the organised/stochastic initial soil moisture condition and table 3 summarises the calculated runoff volumes and peak discharges for the 20 realisations of the initial soil moisture fields. These results highlight three important points:

1. The stochastic feature of the soil moisture distribution is of crucial importance for runoff generation, especially for a relatively small catchment such as the Neuenbürger Pfad. For the storm event simulated here, there is a very large variation in calculated runoff volume ranging from 21 m^3 (or 0.068 mm) to 2826 m^3 (or 9.1 mm) and of peak discharge ranging from 15.3 l/s to 716.7 l/s, see table 3. The duration of the storm runoff ranges between 66 min and 218 min.
2. In general, the stochastic feature of the soil moisture increases the runoff produced, i.e. the runoff from the reference case considering only organised soil moisture heterogeneity (solid line in Fig 7) exhibits a much smaller response than the hydrograph obtained by averaging the 20 hydrographs obtained by using the stochastic initial soil moisture (dashed line in Fig. 7). For the storm event simulated here, the peak value obtained from averaging the 20 stochastic hydrographs (227.7 l/s) is about 3 times higher than the organised hydrograph (75.5 l/s). The difference of the runoff volumes is even more pronounced: 638 m^3 compared to 115 m^3, i.e. the total runoff obtained by averaging the 20 'stochastic hydrographs' exceeds the one obtained by the 'organised hydrograph' by factor 5.5.
3. For the catchment and event reported here, the stochastic component of the soil moisture has a stronger impact on the runoff than the organised component. This is because the difference in calculated runoff is much stronger within the different organised/stochastic soil moisture fields (Fig. 7) than the differences obtained using the different interpolation methods (Fig. 6). However, it has to be stressed that this is due to the specific (dry) feature of the pre-event conditions (July 13, 1992), which showed only a moderate organised but a rather strong stochastic variability. Bronstert (1994) showed an example of wet conditions in the Weiherbach Catchment, where the organised variability was much more pronounced. In general one can expect that the stronger the organised heterogeneity is, the more important it is to have an adequate interpolation technique.

Conclusions and outlook

The case study presented above gives a clear example of how small scale parameter variability can result in non-linear behaviour of a hydrological process. i.e. it has been shown that the amount of surface runoff generated by infiltration excess depends not only on the average initial soil moisture content but also strongly on its spatial variations. Both the organised and the stochastic component of variability are of importance. That means, that the cause for the non-linearity at a certain scale can be due to both the non-linearity of the process-equation *and/or* due to the effects of small scale variability. It also has been shown that accounting for spatial variability results in a general increase of runoff production compared to the results obtained by using averaged values.

However, it has to be emphasised, that this case study presents the results for one specific catchment only. These results are valid for a spatial scale of a few hectares and for rainfall events, where the rainfall intensity is of the same order of magnitude as the average soil infiltration capacity. The soil moisture pattern prior to the summer storm event was characterised by the stochastic variability component, the organised component was much less pronounced. This is in accordance with the investigations of Grayson *et al.* (1997) who found dry catchment conditions being characterised by stochastic moisture pattern, while wet catchment conditions showed a much more organised pattern. Furthermore, at larger scales additional phenomena, such as the impacts of rainfall variability or runoff concentration processes (routing in the channel network), may superimpose the effects discussed in this study.

The specific results obtained from this study can be summarised as follows:

- The variations of the initial soil moisture conditions play an essential role for storm runoff generation at the hillslope and small catchment scale, in particular if infiltration-excess induced surface runoff is of high relevance for catchment runoff. It has been shown that accounting for spatial variability of pre-event soil moisture results in an increase of runoff production compared to averaged values. In the case study reported here, the runoff peak increased by factor 3 and runoff volume by factor 5.5 if the stochastic variability component was included in the soil moisture input data. These results are similar to the study of Binley *et al.* (1989), who found an increase in runoff production with increasing areal heterogeneity of soil conductivity values.

  The modelling study showed that the surface runoff at the scale of a small catchment was very sensitive to changes of the specific stochastic realisation of soil moisture distribution. This is in accordance with the study of Coles *et al.* (1997), who reported an extreme sensitivity of the catchment response to the antecedent soil moisture and soil surface conditions. Also de Roo *et al.* (1996) experienced a high sensitivity of catchment runoff which they related to the variability of hydraulic conductivity.

- In a study investigating the influence of event size, Bronstert (1999)b has shown, that the effects described above are particularly important for events with high (but not extreme) rainfall. Small events (all precipitation infiltrating) cannot show significant space-time variations in runoff production. Extreme events producing a high proportion of infiltration excess will lead to widespread runoff generation, which again reduces variations in total runoff volume. However, events where the rainfall intensity only slightly exceeds the actual infiltration capacity (as in this case study) produce runoff which is very sensitive to fluctuations of the spatial variability of the antecedent soil moisture content.
  This means that for small-scale modelling studies one has to use either high resolution measurement values or input data which show similar stochastic features to those observed in nature.
- For catchment conditions where the stochastic feature of the variability is significant, (which according to Grayson *et al.* (1997) may occur mainly during dry catchment conditions), it is of particular importance to consider the combined organised/stochastic pattern of soil moisture. It is the superposition of systematic and random features of soil moisture which leads to conditions of local saturation, i.e. to the generation of surface runoff. Moreover, the organised component (expressed in higher saturation values in the valley bottoms) increases the "connectedness" of the local runoff generating areas and transfers the generated runoff downslope without significant re-infiltration losses.
  On the other hand, for conditions with a dominant organised soil moisture pattern (which may occur mainly during wet catchment conditions), an accurate description of hydrological responses requires an adequate representation of the deterministic spatial variability (Seyfried & Wilcox, 1995). For such conditions it is important to apply an adequate and refined interpolation technique being capable of accounting for complex spatial trends characteristic of the relevant organisational feature of soil moisture. The derivative of the kriging method (Markov-Bayes-Updating) presented above is a suitable tool to meet this requirement.
- The spatial variability of rainfall was not investigated in this study. The results of Kolle & Fiedler (1995) in the Weiherbach Catchment as well as detailed investigations in other areas show that the spatial variability is of less importance at the spatial scale which has been discussed here (Sprecher, 1988). For larger scales, the realistic representation of the spatial rainfall patterns becomes more crucial, as shown by Woolhiser and Goodrich (1988) and by Faurès *et al.* (1995).

The authors conclude that successful distributed deterministic modelling at the scale discussed here requires very detailed input data (and the corresponding parameterisation) of initial soil moisture content. Similar requirements have to be met for other variables, such as precipitation, soil texture, topography and vegetation features. Averaged input data cannot be recommended. Data of such quality can be provided only in exceptional cases, such as, e.g. from well instrumented and monitored experimental hillslopes or some existing research catchments. Hence, in most applications, one should consider additional stochastic components of the driving forces and the boundary conditions to obtain a realistic picture of the possible hydrologic response of hillslopes and small catchments.

A possible strategy for upscaling in space should not try to generate 'effective' parameters for each scale but try to include the scale dependent effects of the overall spatial pattern (and partly the stochastic features) into the process laws. An upscaling might be achieved more successfully by upscaling the process laws (in our example infiltration and runoff generation) rather than the input parameters. This strategy is similar to the one proposed by Beven (1995), recognising the scale-dependency of any hydrological model structure, where the effects of small scale variability (which becomes sub-scale variability at larger scales) of the investigated processes are to be included in the parameterisation procedure. This sub-scale variability might be represented by a statistical parameter distribution as proposed, e.g. by Famiglietti and Wood (1994).

The effects of sub-scale heterogeneity on surface runoff generation are of particular importance for 'medium' storms where the runoff volume is a small fraction of rainfall. That means that for more intense rainfall events the relative effects of heterogeneity will decrease; this can also can been concluded from the modelling results presented by Grayson *et al.* (1995). Merz and Plate (1997) proposed to characterise the threshold above which the effects of spatial variability might be neglected. This idea (accepting the fact that any individual catchment and even any specific state of the catchment might have its own threshold) can be promising in reaching an appropriate and simplified solution for the modelling of extreme events

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References

Ahmed, A. and de Marsily, G., 1987. Comparison of geostatistical methods for estimating transmissivity using data on transmissivity and specific capacity. *Wat. Resour. Res.*, **23**, 1717–1737.

Bárdossy, A. and Lehmann, W., 1998. Spatial distribution of soil moisture in a small catchment: Part 1: Geostatistical analysis . *J. Hydrol.*, **206**, 1–15.

Beven, K.J. and Kirkby, M.J., 1979. A physically based variable contributing area model of basin hydrology. *Hydrol. Sci. Bull.*, **24**, 43–69.

Beven, K.J., 1995. Linking parameters across scales: subgrid parameterizations and scale dependent hydrological models, *Hydrol. Process.*, **9**, 507–525.

Binley, A., Beven, K. and Elgy, J., 1989. A physically based model of heterogeneous hillslopes. 2. Effective hydraulic conductivities. *Wat. Resour. Res.* **25**, 1227–1233.

Blöschl, G. and Sivapalan, M., 1995. Scale issues in hydrological modelling—a review, *Hydrol. Process.*, **9**, 251–290.

Bronstert, A., 1994. *Modellierung der Abflußbildung und der Bodenwasserdynamik von Hängen*, Report of the Institute of Hydrology and Water Resources Planning, No. 46, University of Karlsruhe, 192 pp.

Bronstert, A., 1999a. Capabilities and limitations of detailed hillslope hydrological modelling. *Hydrol. Process.*, **13**, 21–48.

Bronstert, A., 1999b. Zur Bedeutung von Boden- und Niederschlagseigenschaften für das Auftreten von Infiltrationsüberschuß. *Hydrologie und Wasserbewirtschaftung*, **43**, (accepted)

Bronstert, A. and Plate, E.J., 1997. Modelling of runoff generation and soil moisture dynamics for hillslopes and microcatchments, *J. Hydrol.*, **198**, 177–195.

Coles, N.A., Sivapalan, M., Larsen, J.E., Linnet, P.E. and Fahrner, C.K., 1997. Modelling runoff generation of small agricultural catchments: Can real world runoff responses be captured? *Hydrol. Process.*, **11**, 111–136.

de Roo, A.P.J., Offermans, R.J.E. and Cremers, N.H.D.T., 1996. LISEM: a single-event, physically based hydrological and soil erosion model for drainage basins. II: Sensitivity analysis, validation and application. *Hydrol. Process.*, **10**, 1119–1126.

Eagelson, P.S., 1970. *Dynamic Hydrology*. McGraw Hill, New York, USA, 462 pp.

Famiglietti, J.S. and Wood, E.F., 1994. Multiscale modeling of spatially variable water and energy balance processes, *Wat. Resour. Res.*, **30**, 3061–3078.

Faurès, J.-M., Goodrich, D.C., Woolhiser, D.A. and Sorooshian, S., 1995. Impact of small-scale spatial rainfall variability on runoff modeling, *J. Hydrol.*, **173**, 309–326.

Germann, P.F., 1990. Macropores and hydrologic hillslope processes: Chapter 10 in: *Process Studies in Hillslope Hydrology*, M.G. Anderson and T.P. Burt (eds.), Wiley, Chichester, UK, 327–363.

Grayson R.B., Blöschl, G. and Moore, I.D., 1995. Distributed parameter hydrologic modelling using vector elevation data: THALES nad TAPES-C, Chapter 19 in *Computer Models of Watershed Hydrology*, V.P. Singh (ed.). Wat. Resour. Pub., 669–696.

Grayson R.B., Western, A.W., Chiew, F.H.S., and Blöschl, G., 1997. Preferred states in spatial soil moisture patterns: Local and nonlocal controls, *Wat. Resour. Res.*, **33**, 2897–2908.

Gelhar, L.W., 1984. Stochastic Analysis of Flow in Heterogeneous Media, in *Fundamentals of Transport Phenomena in Porous Media*. Edited by J. Bear & M.Y. Corapcioglu. NATO ASI Series E—No. 82. Martinus Nijhoff, Dordrecht (NL), 673–717.

Kibler, D.F. and Woolhiser, D.A., 1970. *The kinematic cascade as a hydrological model.* Hydrology Papers, 39, Colorado State University, Fort Collins, 27 pp.

Kolle, O. and Fiedler, F., 1995. Messung und numerische Simulation der Energie- und Feuchtebilanz der Bodenoberfläche. Intermediate report 1994 of the BMBF-project 'Prognosemodell für die Gewässerbelastung durch Stofftransport aus einem kleinen ländlichen Einzugsgebiet", Institute of Hydrology and Water Resources Planning, University of Karlsruhe, 43–67.

Lehmann, W., 1995. Anwendung geostatistischer Verfahren auf die Bodenfeuchte in ländlichen Einzugsgebieten, Report of the Institute of Hydrology and Water Resources Planning, No. 52, University of Karlsruhe, 174 pp.

Matheron, G., 1971. The theory of regionalized variables and its applications. *Les Cahiers du Centre Morphologie Mathématique*, Fasc. 5., 211pp.

Merz, B. and Plate, E.J., 1997. An analysis of the effects of spatial variability of soil and soil moisture on runoff. *Wat. Resour. Res.*, **33**, 2909–2922.

Nielsen, D.R., Biggar, J.W. and Ehr, K.T., 1973. Spatial variability of field-measured soil-water properties, *Hilgardia*, **42**, 215–259.

Plate, E.J. (Ed.), 1992. Final report of the 1st phase of the BMBF-project 'Prognosemodell für die Gewässerbelastung durch Stofftransport aus einem kleinen ländlichen Einzugsgebiet', Report of the Institute of Hydrology and Water Resources Planning, No. 41, University of Karlsruhe, 548pp.

Schiffler, G.R., 1992. Experimentelle Erfassung und Modellierung der Infiltration stärkerer Niederschläge unter realen Feldbedingungen, Report of the Institute of Hydrology and Water Resources Planning, No. 40, University of Karlsruhe, 217 pp.

Seyfried, M.S. and Wilcox, B.P., 1995. Scale and the nature of spatial variability: Field examples having implications for hydrologic modeling, *Wat. Resour. Res.*, **31**, 173–185.

Sharma, M.L. and Luxmoore, R.J., 1979. Soil spatial variability and its consequences on simulated water balance, *Wat. Resour. Res.*, **15**, 1567–1573.

Sprecher, K., 1988. *Die Variabilität des Regens und ihre Wirkung auf den Abfluß mit Konsequenzen für das Meßnetz der Großer Runs*, Dissertation, ETH, Zürich, 248pp.

Western, A.W., Blöschl, G., Willgoose, W.R., McMahon, T.A. and Grayson, R.B., 1996. Field investigations of spatial organisation of soil moisture in a small catchment, in *Water and the Environment*, 23rd Hydrology and Water Resources Symposium, 21–24 May, The Institution of Engineers Australia, Nat. Conf. Pub. 96-05, Hobart, Australia, 547–553.

Woolhiser, D.A., 1986. Sensitivity of calculated peak runoff rates to rainfall sampling frequency, in M.E. Moss (ed.) *Integrated Design of Hydrological Networks*: IAHS Publ., No. 158, IAHS Press, Wallingford, UK, 161–171.

Woolhiser, D.A. and Goodrich, D.C., 1988. Effects of storm rainfall intensity patterns on surface runoff, *J. Hydrol.*, **102**, 335–354.

Woolhiser, D.A., Smith, R.E. and Giraldez, J.V., 1996. Effects of spatial variability of saturated hydraulic conductivity on Hortonian overland flow, *Wat. Resour. Res.*, **32**, 671–678.

Zhu, H. and Journel, A.G., 1993. Formatting and Integrating Soft Data: Stochastic Imaging via the Markov-Bayes Algorithm. In: *Geostatistics Tróia '92*, A. Soares (ed.), Kluwer, Dordrecht, 1–12.